Bharatanatyam single hand gesture has 32 different gestures. We need to represent those gestures using ontology. The basic structure and orientation and position/direction of the hand will be important and necessary to be modelled. I have defined a palm co-ordinate system to represent the basic structure and direction of the fingers. Main advantage of this approach is the direction of the fingers will be independent w.r.t. global co-ordinate system. The relationship between hand and arm (how both are oriented) can be defined using the wrist joint system. The movement of the gesture system (hand and arm) during dance can be represented using three joints of our hand system. I am avoiding the global co-ordinate system because of the complexity of representing human limb parts using the system. Also, global co-ordinate system cannot successfully represent a body part movement and its actual trajectory.

The palm co-ordinate system is defined as follows.

Co-ordinate system definition

We need to define a three dimension co-ordinate system for the palm. X-axis is defined on the palm surface normal to the fingers. Y-axis is defined straightforward on the palm surface between middle and ring fingers of the hand. Z-axis is defined as perpendicular to the palm. The origin will be in the middle of the middle finger and ring finger-joint.

Direction of the co-ordinate system

Direction to the positive X-axis is named as **Right (toward thumb)**.

Direction to the negative X-axis is named as **Left**.

Direction to the positive Y-axis is named as **Outward.**

Direction to the negative Y-axis is named as **Inward**.

Direction to the positive Z-axis is named as **Front**.

Direction to the negative Z-axis is named as **Back**.

It needs to be mentioned that the X-axis direction will be same for both hands. It means Direction towards the thumb will be considered as right for both hands.

As per our hand anatomy, our finger movements are restricted to certain points. For example, our four fingers (other than thumb) can only rotate significantly on XZ plane i.e. w.r.t. Y axis. Those can also rotate on XY plane but in a very small range. But thumb can rotate in any direction.

If a finger is pointed in between positive X and Y axis then it can be represented as **Outward-Right** (If it is closer to Y axis) OR **Right-Outward** (If it is closer to X-axis). It can be extended for other quadrant space also.

For three dimension (for thumb), if the finger is pointed to first octant then it can be presented as **Outward-Right-Front** (i.e. finger is closest to Y-axis then X-axis then Z-axis).

Definition of angle in the co-ordinate system

We will define rotation angle in two-dimension. As the axes direction is already named, angle within a quadrant can be specified using the direction name. For example, in the XY plane, I have to represent one angle in the first quadrant. If it is 0 degree then the direction is written as Right. If it is 90 degree then the direction is Outward. If the angle is in between 0 and 45 then the direction is **Right-Outward**. If it is in between 45 to 90 then the direction is **Outward-Right**. Suppose we need to divide the lower part into two again. Then lowest part (0 to 22.5) can be represented as **Outward-Outward-Right**. Middle part can be represented as **Outward-Outward-Right**. Like this, we can extend same for others also. (This part needs more clarity)

Alignment with C-ordinate Axis

I have defined terminology to represent alignment of fingers with the co-ordinate axes.

The finger aligned with the axis --- **Parallel**

The finger Normal to the axis --- **Normal**

The finger is acute with the axis --- **Lnormal**

The finger is obtuse with the axis --- **Gnormal**

Representation of different finger parts

Human finger joints are fixed. There are three joints in our fingers (except thumb) named as MCP, PIP, DIP. The top of a finger is named as TIP. Hence, we have four important positions i.e. MCP, PIP, DIP, and TIP. Thumb finger has three joints including TIP i.e. MCP, IP, and TIP. Each segment between two joints can be represented as tuple consisting of two joint names. A finger segment has different structural attributes i.e. name, position, direction

Name – Finger name

Position – Represented by joint tuple

Direction – Represented by axis direction

Touch Function Definition

Sometimes, we need to represent the connection between two finger parts. For that, a binary Touch function is defined. The domain and range of the Touch function will be a finger part (it can be finger segment OR finger TIP) i.e. tuple. Hence, signature of Touch function is Touch (segment1, segment2).

Bend Function Definition

Finger can bend at different joints. Also, bend can be performed at different planes. Four fingers except thumb can bend w.r.t. Y axis (Flexion/Extension) and Z axis (Abduction/Adduction). Thumb finger has another special movement called Opposition/Reposition w.r.t. X axis. Now flexion/Extension movement may involves three finger joints bend. Abduction/Adduction is done at MCP joint. Opposition/Reposition is also done at thumb MCP joint.

Flexion/Extension – Each joint rotation has specific range (<https://www.researchgate.net/figure/Range-of-motion-of-finger-flexion_tbl1_265967728>). Now this operation can be defined as a triple of three rotation angles. For example, if a finger is straight to the X axis, then the flexion triple will be <0, 0, 0>. Here, MCP joint angle represent the first element, PIP represent the second, and DIP represent the third. Suppose a finger is vertical to the palm plane i.e. aligned with Z axis, then the flexion triple will be <90,0,0>.

Abduction/Adduction – Expected range of motion is 25 degree. If the abduction happens towards positive Y axis then the angle is considered as positive and vice-versa. Angle between two fingers is computed as difference of abduction angle of both fingers. If two fingers are closed then it means that difference of abduction angle of both fingers is 0.

Opposition/Reposition – It is a movement of thumb. Opposition is produced by combination of flexion and abduction of the thumb at MCP joint. Returning the thumb to its anatomical position is called Reposition.

Degree of Imperfection

As of now, we have a copy-book mathematical description of each hasta mudra where every characteristic are described in crisp manner. In a real life scenario, if we want to classify a mudra to those crisp classes that is not possible without introducing fuzziness to our data. For example, pataaka mudra has all fingers parallel to the Y axis. Question is how much deviation of a finger from the ideal position can be accepted? To attain this question of imperfection, we have introduced a membership function for each finger structure.

The membership function for each finger is calculated as

Σ(finger) = MAXfor all i(diff(αi, βi)/(Maxi - Mini))

Where,

∑ = Membership function

α = Ideal angle

β = Current angle

Maxi - Mini = Range of angle of current joint

Note that, the membership value is in between [0,1]. If it is 0 then the finger ideally matches the defined structure.

Hence, each finger position and output of touch criteria belongs to a fuzzy set M where

M = {(y, ∑(y)) | y € (thumb, index, middle, ring, little, touch(.)) }

Now, this membership value will help us to classify a deformed mudra to a particular set based on an acceptance threshold.

Deformation of a finger depends on the joint angles involved in that finger. The joint angle value can change in a certain range. Structure of a finger is dependent on the value of joint angles at certain time. Hence, three joint angles can be considered as free variables or three axes in a co-ordinate system. A point in that co-ordinate system represents a finger structure.

Now a particular mudra contains defined finger positions for each finger. For a particular finger, a mudra can be represented as a co-ordinate in the joint angle co-ordinate system. Now the membership value of a mudra class is maximum (1.0) at that particular co-ordinate. The membership value will decrease if we shift from that co-ordinate. After certain point of shifting, the membership value will be zero. Hence, the membership probability distribution follows Gaussian in the joint angle co-ordinate system. Each mudra position will have a bell curve which tells us the membership value of that mudra. The mean value of a particular Gaussian will be the co-ordinate of the finger position. Initially, we consider that there is no correlation between joint angle variables. The standard deviation for each variable can be assigned with different values based on the acceptance criteria of a particular mudra.